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306, 544, 558, 562

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Current discussion of papers sponsored by the Structural Division is presented as follows:

Number		Page
306	An Engineering Approach to Blast Resistant Design, by N. M. Newmark. (October, 1953. Prior discussion: 373. Discussion closed)	
	Newmark, N. M.	1
544	Examples of Timber Structure Failures, by Michael N. Salgo (November, 1954. Prior discussion: 589. Discussion closed)	
	Gloss, Ralph H.	3
558	An Influence Line Analysis for Suspension Bridges, by David J. Peery. (December, 1954. Prior discussion: none. Discussion closed)	
	Sih, Nan sze	7
	Karol, Jacob	8
	Palmbaum, H. M.	13
	Franciosi, Vincenzo	15
562	Moments and Deflections in Steel Grid Bridge Floors, by H. J. Greenberg. (December, 1954. Prior discussion: none. Discussion closed)	
	Chu, Kuang-Han	29
	Chang, F. K.	29

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Discussion of
"AN ENGINEERING APPROACH TO BLAST RESISTANT DESIGN"

by N. M. Newmark
(Proc. Sep. 306)

N. M. NEWMARK,[1] M. ASCE.—The writer expresses his thanks to Dr. Fraenkel for his comments and for the further explanation of some of the more difficult points involved in design for dynamic loading.

Too often in structural engineering, analysis and design procedures are confused. In many statical problems the confusion is not serious although it leads to wasted time and consequent inefficiency. However, the calculation of dynamic deflections of a structure are affected to such a major degree by uncertainties in the parameters defining the structure and loading that an accurate calculation is an entirely academic task. In order to arrive at any solution of the practical engineering problem, one must decide beforehand which quantities are negligible and which are not. The writer has attempted to do this in his paper. Many items were omitted entirely from consideration because of their negligible influence on the result. It is hoped that the necessary condensation of the paper did not make some of the points too abstruse.

Because of Dr. Fraenkel's long association with dynamical problems and structural behavior, his comments are greatly appreciated since they stem from his experience and knowledge of the subject.

The further discussion given by Dr. Fraenkel on the negative phase of blast loading adds to the value of the paper. Another reason why the negative phase can ordinarily be neglected is the fact that the sequence of events in most cases involves a maximum response before the negative phase starts. For very small bombs acting on structures with a long period of vibration this would not be true. In such cases the negative phase of the blast acts to reduce the effect of the loading and this reduction is important enough to take into account. However, for most cases of practical importance, and for atomic bombs in general, either the negative phase of the blast comes after the maximum response, or the reduction in the response of the building produced by the negative phase is so slight that it is not important to take into account.

The problem of vertical loads is an important one as indicated by Dr. Fraenkel. The added vertical load produced by the vertical component of blast forces on the roof of a resistant building may affect the response. Some account of the added vertical load may be taken by consideration of an increased dead load equivalent to the added vertical force. Further studies of these points are necessary, however.

Another topic about which very little is known concerns buildings with moderate amounts of openings. Dr. Fraenkel's discussion, and the studies that he refers to, will be helpful in consideration of this problem. However, major uncertainties still exist in our knowledge of the loading on structures with openings ranging from ten to fifty per cent of the frontal area.

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The writer has had occasion to use the procedures described in the paper in a number of problems and has found that even for relatively complicated cases the simple procedures are sufficiently accurate for practical purposes. It is important to emphasize the fact that the procedure given in the paper permits one to treat the problem of dynamic structural design from the point of view of an equivalent static load. The major innovation of the paper lies in the method of computing the equivalent static load, and in the fact that the equivalent load is different for the different elements of a structure, being a function of easily definable structural parameters such as the period of vibration, the permissible plastic or inelastic deflection, and the duration of the loading.

Discussion of
"EXAMPLES OF TIMBER STRUCTURE FAILURES"

by Michael N. Salgo
(Proc. Sep. 544)

RALPH H. GLOSS,[1] A.M. ASCE.—The author's review of wartime mistakes in timber construction serves as a helpful reminder of the results which can occur when designers, fabricators, contractors, and maintenance crews are literally forced to use a material with which they are not familiar, or are in such a hurry that there is no time for proper application of what they do know.

The irony of World War II experience is that the mistakes and troubles were not a basic fault of the material and with few exceptions could have been avoided or certainly minimized, and yet the limited number of unsatisfactory experiences of that period may have adversely affected some persons' opinions of timber construction. Others are undoubtedly influenced by lack of knowledge of the true cause of the trouble.

Each of the author's examples seems to the writer to fall in the class of being avoidable, without need for any drastic change in design and construction practice with respect to truss type, method of framing or fastening. Perhaps unintentionally the paper, particularly some parts of the conclusions, may be construed as placing the blame largely on truss types or fastening means rather than on the errors of design, fabrication and maintenance which the author has recognized. Thus the writer feels some further discussion of the paper is in order.

The author has clearly acknowledged that the overall performance record of timber trusses built during World War II has been good, considering the conditions under which the structures were designed and built.

The parallel chord or flat truss with timber connectors were the popular style truss, due to their adaptability to fast field fabrication which was the order of the day, and unfortunately the troubles which resulted are thereby blamed by some on the truss type or on the fastening used rather than on the conditions of the times. It matters not to some who have witnessed or learned of such troubles, that they could and would have been avoided in normal times; just as such troubles were avoided prior to World War II and since then. The same general types of construction are being satisfactorily used today.

The writer does not feel that results would have been substantially different if other truss types, such as the bowstring, had been employed and subjected to the same abuses. With reference to the specific failures discussed in the paper, the general causes of failure cited would likely apply to any type of truss or certainly be avoidable.

The "Act of God" failure of the LTA hangars is certainly unrelated to type of framing or fastening.

The design inadequacy and cross-grain tension failure of the storehouse building would seem independent of truss type. At least one large warehouse project with which the writer is familiar involved failure of bowstring trusses

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due to the same type of cross-grain tension, but at eccentric web member connections which are customarily used.

The deflection of the shipyard trusses was to be expected with unseasoned lumber. The writer has personally seen ring grooves cut grossly oversize for easy installation which could have accounted for some of the deflection. Trusses too frequently were not properly cambered to begin with. The cutting off of web members violating basic design principles does not require comment. It is regrettable that sufficient design clearance was not provided in order that the more than expected deflection would not have hindered use of the building.

The test of aircraft plant trusses is of interest and the better results of the tight trusses are in line with experience. The tight test truss however has no parallel in practice since seasonal variation in lumber moisture content causes all truss bolts to be slightly loose at some times during the year. Basic tests on which connector values are based, however, used joints with bolts loosened one-half turn to minimize the unreliable friction present in a tight joint. It is not clear from the paper whether the test failures were due to joint or wood failure (fig. 15 appears to be a wood failure). It would be of interest to know whether adequate ratios of test to design loads were obtained even with the loosened joint truss.

The troubles with the shipyard multi-story building resulting from deficiency in design, construction and maintenance would not seem restricted to flat trusses. The discussion of secondary stresses is of particular interest. The writer recognizes the presence of secondary stresses but doubts the validity of most computations to determine their magnitude, since such computations involve too many critical assumptions with respect to joint fixity. The same elasticity, yield, or creep, whatever it may be called, which permits more than expected deflection in unseasoned lumber trusses, and thus theoretically sets up large secondary stresses, also acts to relieve the secondary stress. If the overload or excessive deflection occurred quickly, a secondary stress analysis would have more validity but the writer does not feel it applicable for the gradual deflection increase which is usually the case.

An interesting comparison to secondary stress computation for trusses is the theoretical computation of bending stress set up in one laminate of a curved laminated member which is bent to radius. Theoretically unit bending stresses of many thousand pounds may be created by such bending but there is adequate evidence that such stresses are quickly relieved. Similarly an unseasoned beam in bending takes a permanent set which, on the same reasoning as the secondary stress truss analysis, might be interpreted as setting up high secondary stresses, whereas the permanently "set" beam is just as strong as originally and perhaps stronger by reason of the seasoning.

The writer is in agreement with the author's general conclusions regarding good practice to reduce deflection, particular care in use of unseasoned timber subject to rapid drying conditions due to heat and/or low humidity, and the need for good design and maintenance practice.

The generally good experience with peacetime structures and with most wartime structures does not, in the writer's opinion, justify the author's concern over secondary stresses or eccentricities inherent in lapped joints (not confined to timber connectors). Although the multiple or split chord flat or pitched trusses are not as efficient as the bowstring for balanced loading, they are quite satisfactory where designed and built to accepted standards, and need not be arbitrarily restricted in use.

The writer is familiar with restrictions proposed several years ago for trusses other than bowstring, very similar to the restrictions reported by the author. However, the proposed restrictions with which the writer is familiar were dropped, after thorough consideration, at least a year ago as being unnecessary and unrealistic.

Good performance of timber trusses is the result of proper (1) design, (2) fabrication, and (3) maintenance where necessary; it is not restricted to any one type of truss, type of framing or type of connection. Where good practice is violated in any one of these three general fields of work, trouble may be expected—with other materials as well as with timber.

Specifically commenting on the reported proposed restrictions:

1. Restriction of span is no cure for poor practice—larger spans can be just as satisfactory as smaller spans, or conversely smaller spans can be just as troublesome.
2. Length of wood on a bolt is a measure of need for tightening but not of adequacy. There is no evidence that the writer knows of that joints with bolts or other fastenings are less in need of tightening than those with timber connectors. The amount of shrinkage for equal thickness would be the same in a bowstring as in any other type of truss, and there would be the same general reduction in joint strength and increase in deflection for all truss types if joints were not tightened.
3. Inspection and maintenance if necessary are recommended for all types and spans of heavy trusses built of unseasoned lumber, including the bowstring. Freedom from maintenance is very little, if at all, related to span. The writer has not seen any conclusive evidence that only a certain type of truss, system of framing, or type of fastening has a monoply on freedom from maintenance; most trusses are maintenance free except for possible bolt tightening. Any loose joint should be tightened, any member structurally damaged by seasoning needs repair, regardless of the type of truss in which they occur.
4. As far as the writer knows, no conclusive evidence is available to warrant restriction on number of bolts with or without connectors in a joint, and there is little reason to suspect that bolts alone would be substantially different than connectors. The writer is familiar with data for bolts with connectors which indicate three or four bolts with connectors are superior to two bolts with connectors. The same data indicate that recommended reductions in design loads for unseasoned lumber are adequate.

Discussion of
"AN INFLUENCE LINE ANALYSIS FOR SUSPENSION BRIDGES"

by David J. Peery[1]
(Proc. Sep. 558)

NAN SZE SIH,[2] A.M. ASCE.—The author has used the concept of a fictitious beam in tension in his analysis of the suspension bridge by the deflection theory and the assumption of superposition. This gives the misleading concept of the function of the stiffening truss and the basic principle of deflection theory, since the deflection theory applies to the suspension bridge without stiffening truss as well. However, if η is substituted by $\eta_p + \eta_h$ as defined by the author into the basic equation, the following two equations will result;

$$EI\frac{d^2\eta_p}{dx^2} = M_o - (H_w+H)\eta_p \qquad EI\frac{d^2\eta_h}{dx^2} = -Hy - (H_w+H)\eta_h$$

where y is assumed as parabolic, which has the similar expression as the bending moment of a simple beam. All the results as given by the author can also be obtained by integration from the above relations.

The same is true in the influence line of H for the cable, which can also be obtained by the work and energy concept, and take note that

The convergence of the equation for H in the deflection theory is rapid, usually a second trial will give sufficiently accurate result. This is also illustrated by the author in obtaining the H by interpolation from two assumed cl values. However the accuracy of H for different loading conditions obtained by interpolation from the same assumed cl values may not be the same.

The graph shown in Fig. 1 is drawn by the writer, which gives the H for a bridge with hinged stiffening truss, each curve gives the H due to an advancing live load in the center span starting from a certain point in the span. The Graph was made while computation is progressing, which was then used to predict other trial values, and also serves as a check of the consistency of the results. From Fig. 1, it can be seen that H varies practically linearly only within certain range of loading.

The H for the cable is the only redundant in the analysis: after H is found, to find the bending moment of the stiffening truss is rather simple. In using the deflection theory equations, simplification can be made by omitting certain terms in the equation by inspection, for example, for the moment at point A under the loading as shown;

$$\frac{8f}{l^2}\int_o^l \eta_p\,dx = \eta_h$$

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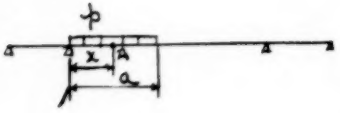


$$C_1 = \frac{p}{2Hc^2}\,\frac{e^{c\ell(1-a)} + e^{-c\ell(1-a)} - 2e^{-c\ell}}{e^{c\ell} - e^{-c\ell}} - \frac{8f}{\ell^2}\,\frac{1}{1+e^{c\ell}}\,\frac{1}{c^2},$$

$$C_2 = -C_1 + \frac{1}{c^2}\left(\frac{p}{H} - \frac{8f}{\ell^2}\right), \quad M_x = -H\left[C_1 e^{cx} + C_2 e^{-cx} - \frac{1}{c^2}\left(\frac{p}{H} - \frac{8f}{\ell^2}\right)\right]$$

if a is small in comparison to the span L, then the equation for moment at A may be written as follows:

$$M_x = -H\left[\frac{1}{c^2}\,\frac{1}{2}\,\frac{p}{H}\,e^{-c(a-x)} - \frac{1}{c^2}\left(\frac{p}{H} - \frac{8f}{\ell^2}\right)\left(1 - e^{-cx}\right)\right]$$

The values of e^{cx} can be interpolated from a table of values of $e^{kc\ell}$.

To find the maximum value of moment at a point, moments are obtained for different length and position of loading, and a curve can be drawn to determine the loading for maximum moment, such as the one shown in Fig. 2.

Similar procedure can be used for all other analysis for the suspension bridge by deflection theory, which can simplify the analysis, and the effect of such simplification is always known. While on the other hand, can the effect of the assumption of superposition as suggested by the author be illustrated by few typical example? It will be useful if effect of this assumption can be expressed analytically.

JACOB KAROL,[1] M. ASCE.—The use of influence lines in the analysis of suspension bridges by the deflection theory is shown in this excellent paper to be both feasible and accurate. The procedure is similar to that used in the analysis of indeterminate structures and should be readily understood by bridge designers. The writer developed a similar procedure in his doctoral thesis,(7) except that a single influence line for cable tension was determined for a value of H based on dead load plus one-half live load on all spans. For calculating the maximum live load moment or shear at a particular point, a loaded length was assumed and the value of H was determined. The moment or shear was then found by substitution in the basic equation of the deflection theory using the final value of $c\ell$. Dimensionless tables for the slopes and deflections at the tenth points of the span and areas under the deflection curves were given in the thesis for integral values of $c\ell$ from 0 to 10 for concentrated and partial uniform loads and for end moments to facilitate analysis. Stiffness and carry-over factors and fixed end moments were also given for integral values of $c\ell$ from 0 to 10, making possible the use of moment distribution in calculating the moments and shears in continuous stiffening trusses. For the same continuous structure analyzed by the author, the moments were generally within 1 % of those determined by Steinman,(2) thus indicating that the use of a single influence line for H gives results accurate enough for final analysis.

With this in mind, it will now be shown that computation of only one set of influence lines is necessary in the author's procedure as well. Simple formulas will be presented for determining the necessary corrections to the final values of $c\ell$.

1. Principal Design Engr., Howard, Needles, Tammen and Bergendoff, Kansas City, Mo.

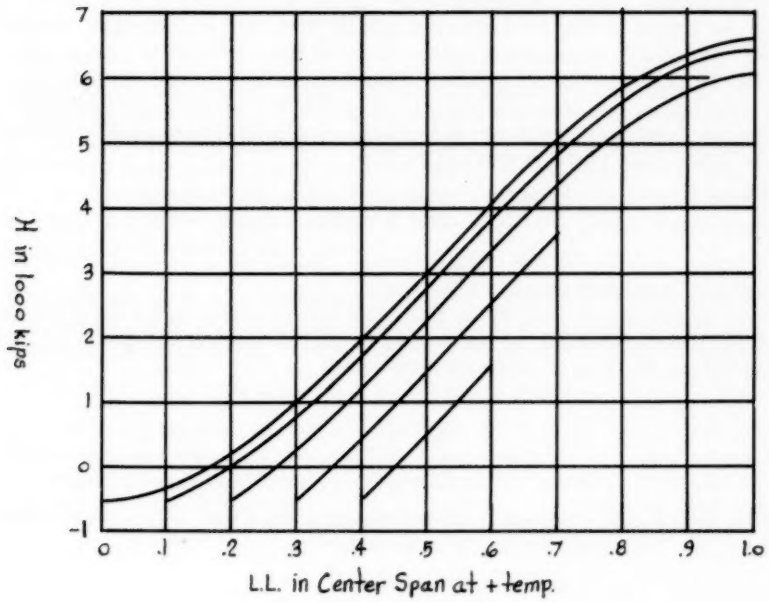
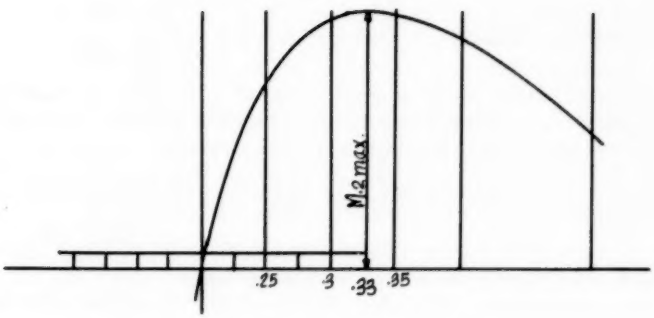


Fig. 1.



Moment at .2 in Center Span

Fig. 2

Let us first consider the influence line for horizontal cable tension for the bridge with hinged stiffening trusses. Assume that this has been calculated for $c\ell = 7.0$ in the main span and $c_1\ell_1 = 3.5$ in the side spans. If the influence line for H for another set of $c\ell$ values is assumed to be affinely related to the known curve, its ordinates will be directly proportional. Hence, it is only necessary to calculate the ordinate at the center of the main span using Eq. 13 to determine this ratio. This applies to the cable tension due to live load only. For temperature, the cable tension is inversely proportional to the D values for the two sets of $c\ell$. Applying this method to the results in Table 3 for $c\ell = 6.0$ and $c_1\ell_1 = 3.0$,

For live load on 0.4 span, H = 339.0 x 1.0333 = 350.3 kips
Temperature $\nearrow 60^{\circ}$, H = -68.7 x 0.804 = -55.2
295.1 kips

The author's computed value is 294.0 kips.

The cable tension for any value of $c\ell$ can also be determined directly from the elastic theory. Studies by the writer indicated that the cable tension due to live load on the main span is approximately

$$H = H_o \times \frac{K}{K + c\ell} \tag{27}$$

where H_o is the cable tension from the elastic theory and $K = 57.5 \div (1 - p/w)$. Similarly, the cable tension due to temperature is

$$H_t = H_{o_t} \cdot [1 + 0.1 (c\ell)^2] \cdot \frac{K}{K + c\ell} \tag{28}$$

where H_{o_t} is the cable tension from the elastic theory. For the loading given above, $H_o = 396.5$ kips, $H_{o_t} = 13.7$ kips and $K = 43.0$. Correcting this to the final value of $c\ell = 6.70$,

For live load on 0.4 span, H = 396.5 x 0.865 = 343.0 kips
Temperature $\nearrow 60^{\circ}$, H = -13.7 x 5.49 x 0.865 = -65.0
278.0 kips

The formulas given above can also be used to correct the cable tension from a known set of $c\ell$ values to another set of $c\ell$ values. Correcting the assumed known values for $c\ell = 7.00$ to the final value of $c\ell = 6.70$,

For load on 0.4 main span, H = 339.0 x 1.006 = 341.0 kips
Temperature $\nearrow 60^{\circ}$, H = -68.7 x 0.9305 x 1.006 = -64.2
276.8 kips

This practically checks the author's value in Table 3.

We will next consider the value of the moment at the 0.2 point in the main span for the given loading. The influence line for moment for another set of $c\ell$ values is not affinely related to that for $c\ell = 7.0$ and $c_1\ell_1 = 3.5$; hence, another method of correction must be used.

Assume that the stiffening truss has zero deflection and moment at the end of the loaded length. Hence, the loaded segment is equivalent to a simple beam. The rate of change of moment for a given span, x, with uniform load to the change in cx is

$$\frac{dM}{d(cx)} = M\left(\frac{-2}{cx} + \frac{1+\cosh 0.5cx}{\sinh cx}\right)\Delta cx$$

$$\approx M\left(\frac{-2\,\Delta cx}{cx - 0.3 + \pi^2/cx}\right) \qquad (29)$$

For greater accuracy the average value of cx should be used in Eq. 29. The correction is applied to the total moment due to live load and temperature.

The moment will be calculated for the final value of $c\ell = 6.70$ since it will be the result used in design. For $c\ell = 7.00$, cx = 2.80. For $c\ell = 6.70$, cx = 2.68. The average value of cx = 2.74 and the correction factor by Eq. 29 is 0.0397 M. Hence, the final moment is

$$M = 1.0397 \times 8046 = 8365 \text{ ft-kips}$$

It is interesting to compute the final moments using the values for $c\ell =$ 6.00 as known. For $c\ell = 6.00$, cx = 2.40. The average value of cx = 2.54 and the correction factor by Eq. 27 is -0.0913 M. Hence, the final moment is

$$M = 0.9087 \times 9189 = 8351 \text{ ft-kips}$$

Both results are in close agreement with the value of 8371 ft-kips obtained by the deflection theory.

A similar correction formula can be written for shears. The rate of change of shear for a given span, x, with uniform load to the change in cx is

$$\frac{dV}{d(cx)} = V\left(\frac{-1}{cx} + \frac{1}{\sinh cx}\right)\Delta cx$$

$$\approx V\left(\frac{-\Delta cx}{cx - 1 + 6.7/cx}\right) \qquad (30)$$

It should be noted that the loaded length is used as the span for positive moments and shears, while the unloaded length should be used for negative moments and shears. The corrections for shear at intermediate points of the span on this basis are approximate but satisfactory, since shears are not usually critical in the design of suspension bridges.

Correction formulas for cable tension for the moments and shears in continuous stiffening trusses would be more complicated than those developed above. However, because of the greater amount of calculation involved in determining the influence lines, even an approximate method of correction would be desirable.

The influence line for horizontal cable tension for another set of $c\ell$ values may be assumed affinely related to that calculated for $c\ell = 7.0$. For live load from the left tower to $x = 0.41\ell$, H = 350 kips for $c\ell = 7.0$. For $c\ell = 6.0$ and $c_1\ell_1 = 3.0$, corrections will be made using Eqs. 27 and 28 even though they apply only to hinged stiffening trusses. Hence, the approximate cable tension for these $c\ell$ values is

For live load on 0.41 span,	H = 350 x 1.020 =	357.0 kips
Temperature $\nearrow 60^\circ$ H = -119.9 x 0.780 x 1.020 =		-95.3
		261.7 kips

By interpolation, the value of H for $c\ell = 6.632$ is approximately 242 kips.

Using this value of H, the total cable tension is 3909 kips, for which the computed $c\ell$ is 6.633. This is a good check on the accuracy of the proposed method, despite the approximations involved.

The correction for the moment at 0.2ℓ in the main span is complicated by the fact that there is a negative moment at the left tower and a positive moment at the right tower for this loading, so that the length to be used for the equivalent simple span is not apparent. It will arbitrarily be assumed as the loaded length, 0.41ℓ. For $c\ell = 7.00$, $cx = 2.87$. For the final value of $c\ell = 6.632$, $cx = 2.72$. The average value of $cx = 2.795$ and the correction factor by Eq. 29 is 0.0497 M. Hence, the final moment is

$$M = 1.0497 \times 6570 = 6896 \text{ ft-kips}$$

This is practically the same as the author's value of 6900 ft-kips.

The designer must use his judgment in estimating the equivalent length to use for correcting the moments at other points in the span. Thus, for the negative moment at the tower, full load on the side span contributes about one-half of the total moment. Hence, a correction based on the side span as the equivalent length should give results that are accurate enough for design. If the influence lines are calculated for a value of H equal to dead load plus one-half live load on all the spans, the corrections will generally be small and the errors necessarily smaller. The assumptions used in design, especially that of uniform moment of inertia, involve greater errors than the correction procedure presented above.

Some discussion of the equivalent uniform moment of inertia to be used in analysis is worthwhile, since the stiffening girders or trusses for longer spans, at least, vary in section along the span. In calculating the influence line for horizontal cable tension, the hangar loads are assumed uniformly distributed along the spans and each span deflects in a single loop. Hence, an equivalent moment of inertia based on equal center deflection for uniform load seems logical for this condition. For partial live load in the main span, that span deflects in two or three loops, and the equivalent moment of inertia is different for each loop. Furthermore, the effect of web deflection is usually ignored in computing the equivalent moment of inertia. Web deflection for the single loops is approximately 5% to 10% of the chord deflection. For the double and triple loops, it would be somewhat greater. However, the effect of ignoring both web deflection and variation in section is compensating, so that the calculated moments and shears based on uniform I are probably somewhat on the safe side. It should be apparent that calculations by the deflection theory for an ideal but hypothetical uniform I do not give the exact moments and shears in the actual structure.

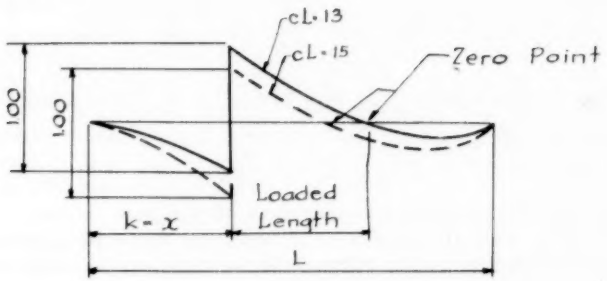
The writer has developed a simple numerical procedure for analyzing suspension bridges by the deflection theory taking account of the variation in I for both hinged and continuous trusses. For the continuous truss investigated by the author, it was found that the maximum negative moment at the tower increased about 10%. For hinged trusses, the maximum moment near the quarter point increased about 3%. These results were based on neglecting the effect of web deflection in computing the equivalent I.

It is hoped that the dimensionless tables prepared by the author will be included in the closure, since his method of analysis is predicated on their availability for ease and rapidity in calculation.

H. M. PALMBAUM,[1] J.M. ASCE.—The author is to be commended for his clear explanation and derivation of the equations for obtaining influence lines in a suspension bridge. However, only the equations for analysis of moments in the stiffening truss were developed. The writer intends to demonstrate how the same type of reasoning can be applied to shears in a simply supported stiffening truss.

To avoid the use of trial and error methods for determining loaded lengths of bridge for maximum shears, values of loaded lengths can be obtained for two extreme values of stiffness. Once the influence lines for shear at a point x are found for two values of stiffness an approximation can be made for some intermediate stiffness. The approximation of straight line interpolation is usual but the shear values do not vary linearly. While the work of setting up the influence line is lengthy and tedious, most of the work, except for a few constants, can be re-used for any other bridge ranging between the available extremes.

The method of variation of the influence line for shear with stiffness is shown in Sketch 1.



Sketch 1.

The physical meaning of Sketch 1 can be explained by an increase in temperature causing an increase in cable sag with a correspondingly lower bridge stiffness. At the lower stiffness value more of the span may be loaded for a maximum positive shear at point x, therefore the zero point for cL = 13, the minimum stiffness to be expected for a particular span, yields a longer critical loaded length than the zero point for cb = 13 does.

Referring to the author's paper, and using an "a" to denote shear equations similar to the author's moment equations, the following equations result:

$$\frac{V}{P}\,(\text{Infl. Line}) = v_p\,(\text{Infl. Line}) + v_h\,\frac{H}{P}\,(\text{Infl. Line}) \qquad (14a)$$

By differentiating equation (8):

$$v_p = \frac{\sinh c(L-k)}{\sinh cL} \cdot \cosh cx \qquad x < k \ .$$

or in exponential form:

1. Engr., Freyssinet Company, Inc., New York, N. Y.

$$v_p = \frac{e^{2(cL-ck)}-1}{2(e^{cL}-e^{-cL})e^{cL-ck}}\left(e^{cx}+e^{-cx}\right) \qquad x<k \tag{8a}$$

By differentiating equation (9)

$$v_p = \frac{-\sinh ck}{\sinh cL}\cdot\cosh c(L-x) \qquad x>k$$

or in exponential form:

$$v_p = \frac{e^{ck}-e^{-ck}}{2(e^{cL}-e^{-cL})}\left[e^{cL-cx}+e^{-(cL-cx)}\right] \qquad x>k \tag{9a}$$

The terms in (8a) and (9a) are independent of physical characteristics and can, when tabulated, be re-used for any suspension bridge of equal stiffness.

$$v_n\cdot\frac{H}{p} = \left[\frac{f^2L}{EI}\cdot\frac{1}{D}\right]\left(-\frac{v_n}{f}\cdot L\right)\left[\frac{-EI}{fL^2}\cdot\eta_n\right] \tag{13a}$$

In equation (13a) the term $\frac{f^2L}{EI}$ is constant for any one bridge and as such is independent of the cL value. The term $\frac{1}{D}$ is a constant for one cL value but for only one bridge. The term $\frac{EI}{fL^2}\cdot\eta_1$ occurs at point k and is a constant factor in the influence line for any one value of stiffness.

By substituting for η_1 in equation (12) the value found in equation (11), integrating and transforming into exponential form, equation (12a) results.

$$D = \frac{L}{E_c A_c} + \sum\frac{8fL^2}{EI}\left(\frac{-8}{c^3L^3}\right)\left[\frac{e^{cL}-1}{e^{cL}+1}\cdot\frac{e^{cL}+e^{-cL}-2}{2} - \frac{e^{cL}-e^{-cL}}{2}+cL-\frac{c^3L^3}{12}\right] \tag{12a}$$

Thus, for one bridge, D varies with cL only, and therefore parts of this term can be re-used in future influence line calculations.

$$\frac{EI}{fL^2}\eta_n = \frac{8}{c^4L^4}\left(\tanh\frac{cL}{2}\sinh cx-\cosh cx+1-\frac{c^2L^2y}{8f}\right) \tag{11a}$$

The terms in equation (11a) show that this factor of equation (13a) can also be re-used in future influence line calculations of the same cL value, provided the sag-length ratio is unchanged.

By differentiating equation (10):

$$-v_n\cdot\frac{L}{f} = \frac{8}{cL}\left(\tanh\frac{cL}{2}\cosh cx-\sinh cx\right) \tag{10a}$$

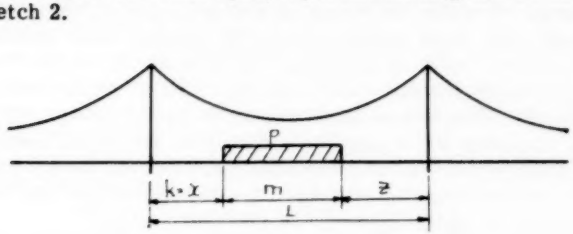
or in exponential form:

$$-v_n\cdot\frac{L}{f} = \frac{4}{cL\cosh\frac{cL}{2}}\left[e^{c\left(\frac{L}{2}-x\right)}-e^{-c\left(\frac{L}{2}-x\right)}\right]$$

By using a tabular form of solution for $-v_n\frac{L}{f}$, v_p and $\frac{EI}{fL^2}\eta_n$ the influence

line for shear, equation (14a), can be developed.

For maximum positive shear values in the main spans from the left end to near the center of the span, a partial loading can be used as shown in Sketch 2.



Sketch 2.

If the influence line is not used, several loaded lengths of span for each of twenty points along the span to be investigated are solved for shear and a curve is plotted for each point showing the variation of shear at that point. An envelope of maximum positive shears is then drawn connecting the maximum value of shear obtained at each of the twenty points along the span. Due to symmetry the number of points can be halved to ten. However, at each one of the points a minimum of three complete results of shear must be obtained for each point.

When the influence lines for shear due to two stiffnesses are known, and the corresponding zero points also, the trials for shear are eliminated.

For example, with a partial loading on the main span and highest positive temperature, the assumed cL value should be at approximately 13.5 for a trial. By linear interpolation between the zero point values for cL = 13 and cL = 15, trial lengths for m and z are obtained. Corresponding to the cL value assumed is a value of H from equation (7). Using deflection theory equations for this loaded length and temperature a value of H is obtained which will be somewhat different from the assumed value, but probably close to the final value. Since the values of H converge rapidly it is a speedy process to solve for new values of c, cL, m and z and find the third value for H. The second trial for H will usually be sufficient and maximum shear values at each point can be immediately solved and the envelope of maximum shears plotted very quickly.

Since the zero points of the influence lines for the longer spans can differ by as much as 2-1/2 % of the length, it seems more reasonable not to use the areas under the influence line. By using the influence line several trials for shear at each point are unnecessary, the shear variation due to partial loading is easily visualized, and design time can be reduced. Furthermore, many of the calculations for the influence line can be re-used as indicated, for future similar work.

VINCENZO FRANCIOSI.[1] —The treatment of D. J. Peery is based on the identity of the equations relating the deflections to the bending moments in both cases: the suspension bridge as well as a simple beam, subjected to an axial force equal to the total horizontal tension H_r, to a uniformly distributed load equal to $-8fH/l^2$, and to the same concentrated loads acting on the suspension bridge.

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From this consideration the possibility is inferred of adding algebraically the stresses producible by the component loadings of a combination of loadings, by unchanged value of the total horizontal tension H_r; this makes possible to draw the influence lines for the stresses; the principal meaning of influence line is however lost. As a matter of fact the applied loads cause the horizontal tension to vary, hence it is impossible to know which position of a given train of loads causes the stresses in a given section of the beam to attain their greatest values. The use of these influence lines is then confined to the evaluation of the stresses for a given load condition, for which purpose it is more convenient simply to draw the influence line of the horizontal tension H; this known, it is an easy matter to determine the deflections and then the bending moments along the entire span of the stiffening truss.[1]

The possibility of adding the stresses due to the components loadings of a given load has already been demonstrated by the writer with reference to arched-rib bridges;[2] this property makes it possible to evaluate the horizontal tension H by means of a solving equation much more expedite than the usual equation of the energy method.[3]

The forces acting on a structure are distinguished in axial and transverse forces, according as they are influenced by the deformation, causing additional stresses, or not. Let us consider the three load conditions of the beam in fig. 1: transverse force P_1, transverse force P_2, transverse forces $P_1 + P_2$; the three load conditions are inclusive of the force H, which is both transverse, because it causes moments $-Hy$ (the moments are considered positive if stetching the inferior fibers), and axial, because it causes moments $-H\eta$ (the deflections η are considered positive if directed downward). In the spirit of the present treatment, forces are always transverse for the tensile and shearing stresses.

The deflections η_1 and η_2 are produced by P_1 and P_2, and η_{1+2} by $P_1 + P_2$, in the structure already deformed by H; the vertical and horizontal deflections are given by the following equations:

$$\begin{cases} EI \cos\alpha \dfrac{d^2\eta_1}{dx^2} = -M_1 - H\eta_1 \\ EI \sin\alpha \dfrac{d^2\xi_1}{dx^2} = -M_1 - H\eta_1 \end{cases} \qquad (1)$$

$$\begin{cases} EI \cos\alpha \dfrac{d^2\eta_2}{dx^2} = -M_2 - H\eta_2 \\ EI \sin\alpha \dfrac{d^2\xi_2}{dx^2} = -M_2 - H\eta_2 \end{cases} \qquad (2)$$

1. G. Albenga, Lezioni di ponti, vol. II page 390, Torino U.T.E.T. 1930. P.B.J. Gravina, Sul calcolo statico dei ponti ad arco sottile con trave irrigidente, Giornale del Genio Civile, Roma, Maggio 1949.
2. V. Franciosi, Il metodo della "linea d'influenza del secondo ordine" per la ricerca della spinta addizionale nei ponti a travata irrigidente, La Ricerca, Napoli, 1951.
3. T. v. Kármán, Mathematical methods in Engineering, McGraw Hill Book Company, New York 1940, page 281.

$$\begin{cases} EI \cos\alpha \dfrac{d^2\eta_{1+2}}{dx^2} = -M_1 - M_2 - H\eta_{1+2} \\ EI \sin\alpha \dfrac{d^2\xi_{1+2}}{dx^2} = -M_1 - M_2 - H\eta_{1+2} \end{cases} \tag{3}$$

where M_1 and M_2 are the bending moments producible by P_1 and P_2 in absence of H.

By adding Eqs. (1) and (2), we obtain:

$$\begin{aligned} EI \cos\alpha \frac{d^2(\eta_1 + \eta_2)}{dx^2} &= -M_1 - M_2 - H(\eta_1 + \eta_2) \\ EI \sin\alpha \frac{d^2(\xi_1 + \xi_2)}{dx^2} &= -M_1 - M_2 - H(\eta_1 + \eta_2) \end{aligned} \tag{4}$$

By comparing Eqs. (3) and (4), we deduce:

$$\eta_{1+2} = \eta_1 + \eta_2$$

$$\xi_{1+2} = \xi_1 + \xi_2$$

which demonstrates the possibility of adding algebraically the effects due to the transverse forces, if the axial forces remain unaltered. By the same limitation the theorem of Betti still works and we can write:

$$P_1\eta_{21} = P_2\eta_{12} \tag{5}$$

Let us consider the two load conditions in fig. 2; η_1 and ξ_1 be the displacements due to the former condition, η_2 and ξ_2 those due to the latter; they are evaluated from the configuration of equilibrium under the own weight w and its corresponding horizontal tension H_w; hence they are caused by the transverse forces P and H (fig. 2a), and H (fig. 2b), in presence of the axial force $H_w + H$, equal in both load conditions. Equation (5) is valid and gives

$$H\xi_{2A} + P\eta_{2P} = H\xi_{1A} \tag{6}$$

Alternatively Eq. (6) can be derived in a direct way.

For that let us consider the bending moments M' caused on the beam by two unit concentrations in A and B respectively, equal but with opposite signs; as well as the moments M'' caused on the same beam by a unit concentration directed along the line of action of P, fig. 3.

The bending moments[4] causing the deflections of fig. 2 are:

$$M_1 = PM'' - H\,y - (H_w + H)\,\eta_1$$

$$M_2 = \quad - H\,y - (H_w + H)\,\eta_2$$

The principle of virtual works, applied to the equilibrated stresses of fig. 3 and to the congruent displacements of fig. 2, yields:

4. Of all the stresses only the bending moments are considered, because, as already said, the forces are here distinguished in axial and transverse only in reference to the bending moments; hence, doubts could arise on the validity of the theorem of Betti.

$$\xi_{1A} = \frac{1}{EI} \int_0^{\ell} M_1 M' \, dx = - \frac{P}{EI} \int_0^{\ell} M'' y \, dx + \frac{H}{EI} \int_0^{\ell} y^2 dx + \frac{H_w + H}{EI} \int_0^{\ell} y \eta_1 \, dx$$

$$\xi_{2A} = \frac{1}{EI} \int_0^{\ell} M_2 M' \, dx = \frac{H}{EI} \int_0^{\ell} y^2 dx + \frac{H_w + H}{EI} \int_0^{\ell} y \eta_2 \, dx$$

$$\eta_{2P} = \frac{1}{EI} \int_0^{\ell} M_2 M'' dx = - \frac{H}{EI} \int_0^{\ell} M'' y \, dx - \frac{H_w + H}{EI} \int_0^{\ell} M'' \eta_2 \, dx$$

Equation (6) can be written then:

$$H^2 \int_0^{\ell} y^2 dx + H(H_w + H) \int_0^{\ell} y \, \eta_2 \, dx - PH \int_0^{\ell} M'' y \, dx - P(H_w + H) \int_0^{\ell} M'' \eta_2 \, dx =$$

$$= - PH \int_0^{\ell} M'' y \, dx + H^2 \int_0^{\ell} y^2 dx + H(H_w + H) \int_0^{\ell} y \, \eta_1 \, dx \, ,$$

and more

$$H \int_0^{\ell} y \eta_2 \, dx - P \int_0^{\ell} M'' \, _2 \, dx = H \int_0^{\ell} y \, \eta_1 \, dx \, ,$$

which can also be written

$$\int_0^{\ell} M_{10} \, \eta_2 \, dx = \int_0^{\ell} M_{20} \, \eta_1 \, dx \, , \tag{7}$$

where M_{10} and M_{20} are the parts of first order of M_1 and M_2. The validity of Eq. (6) is then subordinate to the validity of Eq. (7). It is (derivatives indicated by superscripts):

$$EI \, \eta_1'' = -M_{10} + (H_w + H)\eta_1$$

$$EI \, \eta_2'' = -M_{20} + (H_w + H)\eta_2 \, ,$$

by solving for M_{10} and M_{20} and introducing these values in Eq. (7), we obtain:

$$\begin{aligned} &- EI \int_0^{\ell} \eta_1'' \, \eta_2 \, dx + (H_w + H) \int_0^{\ell} \eta_1 \eta_2 \, dx \\ &= - EI \int_0^{\ell} \eta_2'' \eta_1 \, dx + (H_w + H) \int_0^{\ell} \eta_2 \eta_1 \, dx \, . \end{aligned} \tag{8}$$

By integration, we obtain:

$$\int_0^{\ell} \eta_1'' \eta_2 \, dx = \left[\eta_1' \eta_2 \right]_0^{\ell} - \int_0^{\ell} \eta_1' \eta_2' \, dx = - \int_0^{\ell} \eta_1' \eta_2' \, dx$$

$$\int_0^{\ell} \eta_2'' \eta_1 \, dx = - \int_0^{\ell} \eta_2' \eta_1' \, dx ,$$

and Eq. (8) can then be written:

$$EI \int_0^{\ell} \eta_1' \eta_2' \, dx + (H_w + H) \int_0^{\ell} \eta_1 \eta_2 \, dx$$

$$= EI \int_0^{\ell} \eta_2' \eta_1' \, dx + (H_w + H) \int_0^{\ell} \eta_2 \eta_1 \, dx ,$$

which is an identity. Equation (8) is hence valid and Eq. (6) with it. If H in fig. 2 is caused by P on the cable hinged in A and B, it results $\xi_{1A} = 0$, and Eq. (6) can be written in this case:

$$H\xi_A + P\eta_P = 0 , \tag{9}$$

where the subindex 2 has been omitted for the sake of simplicity.

From Eq. (9) we have

$$H = -P \frac{\eta_P}{\xi_A} ; \tag{10}$$

while for a load extending from abscissa a to b:

$$H = - \frac{1}{\xi_A} \int_a^b p\eta \, dx . \tag{11}$$

Equation (10), directly obtained, is coincident with Peery's Eq. (13); it lends itself, as it will be shown immediately, to the determination of H through a proceeding characterized by very rapid convergence and based on the use of trigonometrical functions. Equation (11) assures that the diagram η, read in the scale in which $\xi_A = 1$, is the influence line of H. Equation (11) is valid however only if the horizontal tension $H_w + H$ is equal in both systems of fig. 2a and fig. 2b; this means that of the infinite lines $\eta(x)$ that can be drawn, each due to an assigned value of H, the only one valid for a given load condition must give, through Eq. (11), the same value for H as it has been drawn for.

Being, as already seen,

$$M = - H y - (H_w + H) \eta ,$$

it results:

$$EI \frac{d^2\eta}{dx^2} = H y + (H_w + H)\eta . \tag{12}$$

With the positions

$$\eta(x) = \sum_{1,3}^{\infty} \eta_n \sin \frac{n\pi x}{\ell} \tag{13}$$

$$y(x) = \sum_{1,3}^{\infty} y_n \sin \frac{n\pi x}{\ell} \ . \tag{14}$$

being

$$y = \frac{4f}{\ell^2} x (\ell - x)$$

there results, by Fourier,

$$y_n = \frac{2}{\ell} \int_0^{\ell} y \sin \frac{n\pi x}{\ell} dx = \frac{32f}{n^3\pi^3} \ .$$

By substituting the values (13) and (14) in Eq. (12) and by equating the coefficients of the sinuses of the same order, there results:

$$\eta_n = - \frac{32 H f}{n^3\pi^3 (\frac{n^2\pi^2 EI}{\ell^2} + H_w + H)} \tag{15}$$

The displacement ξ_A is caused by the moments of first order

$$M_0 = - H y = \frac{4fH}{\ell^2} x (\ell - x) \ , \tag{16}$$

by the moments of second order

$$M^{(2)} = - (H_w + H)\eta = - (H_w + H) \sum_{1,3}^{\infty} \eta_n \sin \frac{n\pi x}{\ell} \ , \tag{17}$$

and by the normal stresses

$$N = \frac{H}{\cos \alpha} \ . \tag{18}$$

To obtain the values ξ_{A1}, ξ_{A2}, ξ_{AN} connected to the stresses (16), (17) and (18), the principle of virtual works can be used. Hence, we refer to the system of equilibrated forces in fig. 3a, connected with the beam bending moments

$$M' = - \frac{4f}{\ell^2} x (\ell - x) \tag{19}$$

and the cable normal stresses

$$N' = \frac{1}{\cos \alpha} \ . \tag{20}$$

For the moments M' there are considered the relative rotations due to the moments M_0 and $M^{(2)}$, for the normal stresses N' the relative axial displacements due to the forces N. We obtain:

$$\xi_{A1} = \frac{1}{EI} \int_0^{\ell} M_0 M' dx = \frac{8 H f^2 \ell}{15EI}$$

$$\xi_{A2} = \frac{1}{EI} \int_0^{\ell} M^{(2)} M' dx = 16 (H_w + H) \frac{f\ell}{EI\pi^3} \sum_{1,3}^{\infty} \frac{\eta_n}{n^3}$$

$$\xi_{AN} = \frac{1}{EA}\int_c N N' \, ds = \frac{H}{EA}\int_c \frac{ds}{\cos^2\alpha} = \frac{H}{EA}\int_0^{\ell} \frac{dx}{\cos^3\alpha} = \frac{HL_s}{EA}$$

where for a parabolic cable is, as known:

$$L_s = \ell + \frac{8f^2}{\ell} .$$

Equation (11) can be written conclusively:

$$H = -\frac{\sum_{1,3}^{\infty} \eta_n S_n}{\frac{8H f^2 \ell}{15EI} + 16(H_w + H)\frac{f\ell}{EI\pi^3}\sum_{1,3}^{\infty}\frac{\eta_n}{n^3} + \frac{HL_s}{EA}} \qquad (21)$$

where

$$S_n = \int_a^b p \sin\frac{n\pi x}{\ell} \, dx .$$

In the presence of a temperature variation $\pm \Delta t$, Eq. (9) is so modified:[5]

$$H\xi_A + P\eta_p = \mp \int_c \frac{H}{\cos\alpha}\omega\Delta t \, ds =$$

$$= \mp H\omega\Delta t \int_0^{\ell} \frac{dx}{\cos^2\alpha} = \mp H\omega\Delta t \, L_t \qquad (22)$$

being for a parabolic cable

$$L_t = \ell + \frac{16}{3}\frac{f^2}{\ell} .$$

From Eq. (22) we obtain:

$$H = -\frac{P\eta_p}{\xi_A \pm \omega\Delta t \, L_t} ;$$

5. V. Franciosi, Sulle linee d'influenza di caazioni, "L'Industria Italiana del Cemento," Roma, Marzo 1950 ___ Note: The generalized principle of Betti, valid in the presence of displacements independent from the external forces (distortions) and with the same limitations as for the classical principle of reciprocity, assures $L_{ab} = L_{ba}$, where L_{ab} and L_{ba} are the mutual works done by the two systems of forces and distortions applied to the same structure, considering as mutual work that done by the external forces of system "a" for the displacements of system "b" and by the internal forces (stresses) of system "a" for the distortions of system "b." In our case it is: (fig. 2)

$$L_{12} = H\xi_A + P\eta_p$$

$$L_{21} = \mp \int_c \frac{H}{\cos\alpha}\omega\Delta t \, ds .$$

Hence, Eq. (21) is modified by adding the term $\omega \Delta t \, L_t$ to the denominator. Use of Eq. (21) offers the advantage of determining H with as many significant digits as desired. In Peery's procedure instead the value to assume for H (Table 3) is obtained by linear interpolation between the values calculated for two values of $c\ell$ (in the example $c\ell = 6$ and $c\ell = 7$), without the possibility of evaluating the error and bettering the approximation. The errors made in calculating the bending moments are therefore twofold; they can be reduced by calculating H through Eq. (21) and by interpolating only to obtain the moments, with the exact value of H as basis. It is to be noted that the same (21) can be used in the study of arched-rib bridges, provided some modifications are introduced. In the expression of η_n (Eq. 15), H_w and H change their signs,

$$\eta_n = \frac{-32\, H\, f}{n^3 \pi^3 \left(\frac{n^2 \pi^2 k}{\ell^2} - H_w - H\right)}$$

where $k = E_t I_t + E_a I_a \cos \alpha$, the being assumed as invariable.

The sign of the term $\omega \Delta t \, L_t$ in Eq. (14) also varies. The term A in Eq.(12) is substituted by the mean value A_m between springer and crown.

Here follows a numerical example, in the case of an arched-rib bridge with the following characteristics

$\ell = 43.2$ m
$f = 5.2$ m
$E = 2.1 \cdot 10^6$ tm^{-2}
$A_m = 0.952$ m^2
$I_a \cos\alpha = 0.00498$ m^4

$I_t = 0.1237$ m^4
$k = 270228$ tm^2
$w = 6.1$ tm^{-1}
$H_w = 273.655$ t
$L_s = 48.207$ m

The load $p = 1$ tm^{-1} acts from abscissa $x = 0$ to $x = \frac{\ell}{2}$. The first value assigned to H is the one producible by p for a three-hinged arch

$$H = \frac{p\ell^2}{16f} = 22.43 \text{ t} .$$

By taking into account the first and second term only of Eq. (12), it is obtained

$$S_n = \frac{\ell p}{n\pi} \left(\cos \frac{n\pi a}{\ell} - \cos \frac{n\pi b}{\ell}\right)$$

$\eta_1 = -0.1063\,847$ m
$\eta_3 = -0.0003547$ m
$\xi_{A1} = 0.0517116$ m
$\xi_{A2} = 0.0135140$ m
$\xi_{AN} = 0.0005408$ m
$\xi_A = 0.0657665$ m

$$S_1 = \frac{p\ell}{\pi} = 13.750986 \text{ t}$$

$$S_3 = \frac{p\ell}{3\pi} = 4.584 \text{ t}$$

$H = 22.268$ t

By recalculating with the new value of H, it follows

$$\eta_1 = 1.0546000 \text{ m}$$
$$\eta_3 = 0.0003522 \text{ m}$$
$$\xi_A = 0.0652639 \text{ m}$$
$$H = 22.245 \text{ t}$$

With another step it results:

$$\eta_1 = 1.0534880 \text{ m}$$
$$\eta_3 = 0.0003518 \text{ m}$$
$$\xi_A = 0.0651930 \text{ m}$$
$$H = 22.246 \text{ t}$$

In the same case the classical equation of energy gives:

a) starting from H = 22.43 t, H = 2.224 t
b) starting from H = 21.892 t, H = 65.083 t
c) starting from H = 22.246 t, H = 22.247 t

It is noted by the developed example that while the equation of the energy does not prove convenient for a calculation through successive approximation because this would diverge, and for its solution special proceedings are necessary (Jakkula) and besides with the use of many digits, Eq. (21) instead can be quickly solved by successive approximation, giving at the second step already the technically exact result requiring the determination of but a few significant digits. The percent error is 0.1% by the first step and 0.005% by the second one. The procedure already shown is easily extensible to the suspension bridges with continuous stiffening truss. Be $\overline{M}$ the value of the moments due to the force H at the towers. Equation (12) is written

$$EI \frac{d^2\eta}{dx^2} = Hy + (H_w + H)\eta - \overline{M}.$$

and being

$$\overline{M} = \frac{4\overline{M}}{\pi} \sum_{1,3}^{\infty} \frac{\ell}{n} \sin \frac{n\pi x}{\ell}$$

Equation (15) is corrected into

$$\eta_n = - \frac{32 H f - 4\overline{M} n^2\pi^2}{n^3\pi^3(n^2\pi^2 \frac{EI}{\ell^2} + H_w + H)} \tag{24}$$

The values of the side span, chosen the origin of coordinates at the supported end, are given by the expression

$$\eta_1 = \sum \eta_{1n} \sin \frac{n\pi x_1}{\ell_1}; \tag{25}$$

from the equation

$$E_1 I_1 \frac{d^2\eta_1}{dx_1^2} = H y_1 + (H_w + H)\ \eta_1 - \overline{M} \frac{x_1}{\ell_1},$$

it is obtained

$$\eta_{1n} \begin{cases} -\dfrac{32\,H\,f_1 - 2\overline{M}\,n^2\pi^2}{n^3\pi^3\left(n^2\pi^2\dfrac{E_1I_1}{\ell_1^2} + H_w + H\right)} & \text{(n odd)} \\ -\dfrac{2\overline{M}}{n\pi\left(n^2\pi^2\dfrac{E_1I_1}{\ell_1^2} + H_w + H\right)} & \text{(n even)} \end{cases}$$

From the equation

$$\left(\frac{d\eta_1}{dx_1}\right)_{x_1 = \ell_1} = \left(\frac{d\eta}{dx}\right)_{x = 0}$$

is found the value of $\overline{M}$

$$\overline{M} = \frac{16}{\pi^2} H \frac{\dfrac{f_1}{\ell_1}\sum_{1,3}^{\infty}\dfrac{1}{n^2\left(n^2\pi^2\dfrac{E_1I_1}{\ell_1^2} + H_w + H\right)} + \dfrac{f}{\ell}\sum_{1,3}^{\infty}\dfrac{1}{n^2\left(n^2\pi^2\dfrac{EI}{\ell^2} + H_w + H\right)}}{\dfrac{1}{\ell_1}\sum\dfrac{1}{n^2\pi^2\dfrac{E_1I_1}{\ell_1^2} + H_w + H} + \dfrac{2}{\ell}\sum_{1,3}^{\infty}\dfrac{1}{n^2\pi^2\dfrac{EI}{\ell^2} + H_w + H}}$$

It is also

$$\xi_{A1} = \frac{8}{15} H \left(\frac{f^2\ell}{EI} + \frac{2f_1^2\ell_1}{E_1I_1}\right) - \frac{2}{3}\overline{M}\left(\frac{f\ell}{EI} + \frac{f_1\ell_1}{E_1I_1}\right)$$

$$\xi_{A2} = 16\,(H_w + H)\frac{f\ell}{EI\pi^3}\sum_{1,3}^{\infty}\frac{\eta_n}{n^3} + 32\,(H_w + H)\frac{f_1\ell_1}{E_1I_1\pi^3}\sum_{1,3}\frac{\eta_{1n}}{n^3}$$

$$\xi_{AN} = \frac{HL_S^*}{EA}$$

where

$$L_S^* = \int_S \frac{dx}{\cos^3\alpha} .$$

The load p_1 acts on the side span, the load p on the central one; by expressing them with

$$P_1 = \sum P_{1n} \sin \frac{n\pi x_1}{\ell_1}$$

$$P = \sum P_n \sin \frac{n\pi x}{\ell}$$

Equation (11) delivers

$$H = -\frac{\frac{\ell}{2} \sum \eta_n P_n + \frac{\ell_1}{2} \sum \eta_{1n} P_{1n}}{\xi_{A1} + \xi_{A2} + \xi_{AN}}$$

$$= -\frac{\sum \eta_n S_n + \sum \eta_{1n} S_{1n}}{\xi_{A1} + \xi_{A2} + \xi_{AN}} .$$

It is started by an arbitrary value of H, for example that producible in the structure statically determined by inserting hinges in the stiffening truss. Through Eq. (27) is obtained $\overline{M}$, through Eq. (24) and Eq. (26) η_n and η_{1n} (stopping in general at n=3), through Eqs. (28), (29), (30) the value of ξ_A; from Eq. (31) is thus obtained a better approximated value for H, from which it is started again for iteration of the procedure.

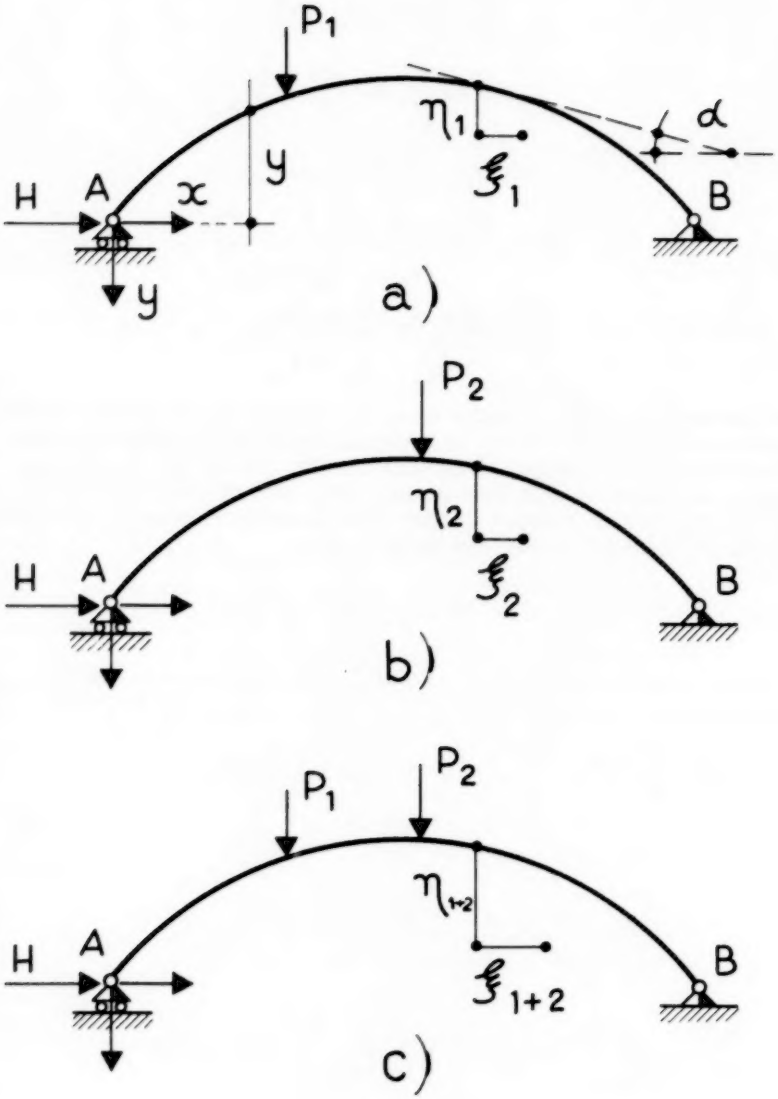


Fig. 1

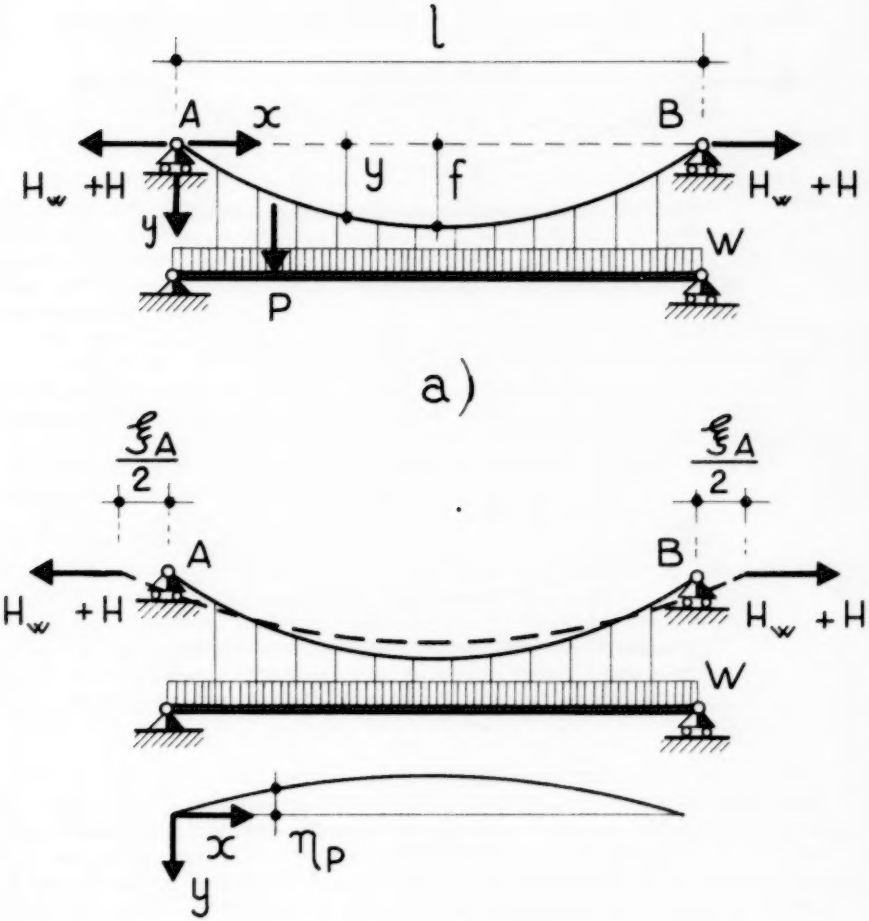
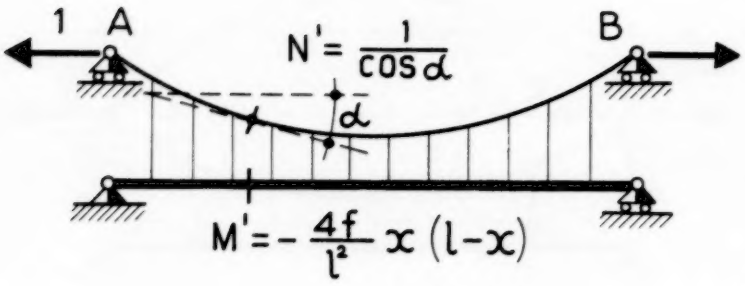
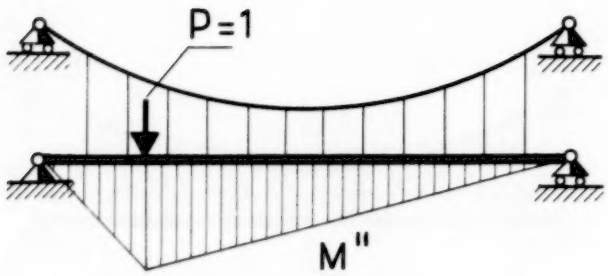


Fig. 2



a)



b)

Fig. 3

Discussion of
"MOMENTS AND DEFLECTIONS IN STEEL GRID BRIDGE FLOORS"

by H. J. Greenberg
(Proc. Sep. 562)

KUANG-HAN CHU,[1] A.M. ASCE.—This paper presents some valuable data for designing steel grid bridge floors. However, the approximate formulas presented seem to be too complicated for routine design purpose. The formulas suggested by the writer are shown below. A comparison of the values thus obtained with those scaled from author's curves is given in the accompanying Tables.

The table for filled grid floors also lists values from Eq. (A) for S>7 ft. for comparison. The moments obtained from Eq. (B) are higher than those given by the author for S>8 ft. However, it should be noted that author's values are based on a single load applied at mid-span while the AASHO formula (Eq. B) may have taken account of the effect of more than one load on span for longer spans. In fact, it should be noted that author's data are subjected to the following limitations:

(1) The wheel load is assumed to be distributed over an area of 15" x 15".
(2) The analysis is made only for two equal span continuous slabs.
(3) The slabs are on unyielding supports of zero width.
(4) For maximum center moments, a single wheel load is applied at the center of one span, and
(5) For maximum edge moments, two equal wheel loads are applied respectively at the centers of adjacent spans.

F. K. CHANG,[2] A.M. ASCE.—The formulas proposed by the author and K. C. Kuo for the calculation of moments and deflections in steel-grid highway bridge floor are very valuable, since the anisotropic property of the floor has been taken into account. The actual problem in the analysis of this kind of floor is rather involved. It is a continuous anisotropic plate on elastic supports with partially restrained edges under concentrated or uniform loading. The author and Kuo assume rigid supports and hinged edge condition. These assumptions are believed to be satisfactory for practical purpose. However, the formulas for maximum moments are based on two continuous spans which are corresponding to the exterior spans, the portion of floor between exterior stringer and the first interior stringer. These formulas are over conservative for interior spans, the portion of floor between interior stringers, which constitute a major part of the floor. Formulas for interior span are also desirable. In actual construction, the interior span length can be made larger than that of exterior span if same floor is used throughout.

1. Structural Designer, D. B. Steinman, Consulting Engr., New York, N. Y.
2. Designer, Ammann and Whitney, New York, N. Y.

For Filled Grid Floors

$$\frac{M_{cx}}{P} = \frac{0.25}{0.65+2.5} \times \frac{1}{\sqrt[5]{r}} \qquad S = 2'-7' \text{ ---- (A)}$$

$$\frac{M_{cx}}{P} = \frac{0.25}{0.45+3.75} \times \frac{1}{\sqrt[5]{r}} \qquad S > 7' \text{ ------ (B)}$$

Note: Formulas A and B for $r=1$ are the same as those given in the AASHO Specifications.

$$\frac{M_{cy}}{P} = 0.75 \frac{M_{cx}}{P} \sqrt[3]{r} \qquad \text{---------- } (C)_{A,B}$$

Note: Formula C_A denotes formula C based on formula A
" C_B " " " " " " " B

$$\frac{M_e}{P} = 0.135 \frac{1}{\sqrt[8]{r}} = \frac{M_{ef}}{P} \qquad S \geq 2.5' \text{ -- (D)}$$

$$\frac{M_e}{P} = \left(\frac{M_{ef}}{P} - 0.05\right) \frac{S}{2.5} + 0.05 \qquad S < 2.5' \text{ ---(E)}$$

For Open Grid Floors

$$\frac{M_{cx}}{P} = \frac{0.25}{1.70} \times \frac{1}{\sqrt[5]{100r}} \qquad S \leq 5' \text{ ---(F)}$$

Note: This formula for $r=0.01$ corresponds with the AASHO Specification for H-15 and grid spacing $b_1 = 2\frac{3}{4}''$ i.e. $\frac{M_{cx}}{P} = \frac{0.2S}{w+2b_1}$ where $w = 1$ in. per ton of H truck. For H-15, $b_1 = 2\frac{3}{4}''$; $w+2b_1 = 1.70$ ft.

$$\frac{M_{cx}}{P} = 0.2\left[\frac{S}{5}+2\right] \frac{1}{\sqrt[5]{100r}} \qquad S > 5' \text{ ---(G)}$$

$$\frac{M_e}{P} = 0.475 \frac{1}{\sqrt[8]{100r}} = \frac{M_{eo}}{P} \qquad S \geq 4.5' \text{ --(H)}$$

$$\frac{M_e}{P} = \left[\frac{M_{eo}}{P} - 0.1\right] \frac{S}{4.5} + 0.1 \qquad S < 4.5' \text{ ---(I)}$$

Filled Grid Floors

$r=\frac{D_y}{D_x}$	Span S ft.	M_{cx}/P			M_{cy}/P			M_e/P		
		FIG 4	Eq A	Eq B	FIG 5	Eq C_A	Eq C_B	FIG 8	Eq D	Eq E
1.0	1.5							0.100		0.101
	2.5	0.128	0.125		0.079	0.096		0.125	0.135	
	5.0	0.190	0.182		0.140	0.137		0.137	0.135	
	7.5	0.229	0.214	0.222	0.172	0.160	0.167	0.138	0.135	
	10.0	0.246	0.235	0.258	0.193	0.176	0.194	0.138	0.135	
	12.5	0.257	0.250	0.285	0.207	0.188	0.214	0.138	0.135	
0.7	1.5							0.103		0.105
	2.5	0.135	0.134		0.070	0.089		0.130	0.141	
	5.0	0.205	0.195		0.138	0.130		0.143	0.141	
	7.5	0.243	0.230	0.238	0.160	0.153	0.158	0.143	0.141	
	10.0	0.261	0.252	0.277	0.179	0.168	0.184	0.144	0.141	
	12.5	0.271	0.268	0.306	0.191	0.178	0.204	0.145	0.141	
0.4	1.5							0.107		0.111
	2.5	0.150	0.150		0.062	0.083		0.136	0.151	
	5.0	0.228	0.219		0.112	0.122		0.150	0.151	
	7.5	0.273	0.257	0.267	0.142	0.142	0.148	0.150	0.151	
	10.0	0.293	0.282	0.310	0.161	0.156	0.172	0.151	0.151	
	12.5	0.306	0.300	0.342	0.173	0.166	0.190	0.152	0.151	
0.2	1.5							0.110		0.119
	2.5	0.168	0.172		0.050	0.076		0.144	0.165	
	5.0	0.261	0.251		0.100	0.110		0.160	0.165	
	7.5	0.312	0.295	0.306	0.125	0.130	0.135	0.162	0.165	
	10.0	0.340	0.324	0.356	0.143	0.142	0.156	0.164	0.165	
	12.5	0.355	0.345	0.394	0.157	0.152	0.173	0.165	0.165	
0.1	1.5									
	2.5	0.190	0.199		0.044	0.069				
	5.0	0.295	0.289		0.085	0.101				
	7.5	0.359	0.340	0.353	0.110	0.118	0.123			
	10.0	0.392	0.374	0.410	0.129	0.130	0.143			
	12.5	0.415	0.399	0.470	0.143	0.139	0.164			

Open Grid Floors ($S/D_x = 0.05$ $S = H/D_x$)

$r=\frac{D_y}{D_x}$	Span S ft	M_{cx}/P			Span S ft	M_e/P		
		FIG 6	Eq F	Eq G		FIG 10	Eq H	Eq I
0.01	2.0	0.20	0.235		2.0	0.260		0.267
	4.0	0.47	0.470		4.0	0.410		0.433
	5.0	0.57	0.590	0.60	4.5	0.422	0.475	
	7.5	0.70		0.70	6.0	0.449	0.475	
	10.0	0.80		0.80	8.0	0.464	0.475	
	12.0	0.86		0.88	12.0	0.473	0.475	
0.02	2.0	0.20	0.204		2.0	0.260		0.250
	4.0	0.43	0.410		4.0	0.395		0.400
	5.0	0.51	0.515	0.522	4.5	0.403	0.436	
	7.5	0.62		0.610	6.0	0.425	0.436	
	10.0	0.70		0.695	8.0	0.440	0.436	
	12.0	0.75		0.765	12.0	0.446	0.436	
0.04	2.0	0.20	0.178		2.0	0.260		0.233
	4.0	0.39	0.356		4.0	0.375		0.366
	5.0	0.45	0.446	0.455	4.5	0.383	0.400	
	7.5	0.55		0.530	6.0	0.400	0.400	
	10.0	0.62		0.606	8.0	0.410	0.400	
	12.0	0.66		0.666	12.0	0.417	0.400	

The steel-grid floor as an anisotropic plate can also be solved by numerical method. The author's Eq. 2 can be replaced by a difference equation and the later, in turn, can be replaced by a set of simultaneous equations. The whole problem will then be reduced to the solution of simultaneous equations.

Assume that the imaginary net shown in the writer's Fig. 1 behaves as an anisotropic plate. The value of $\frac{\partial^2 w}{\partial x^2}$ at any net point, say point "0," can be evaluated approximately from the deflections, w's, of its neighbor points.

$$\frac{\partial^2 w}{\partial x^2} \simeq \frac{\frac{w_E - w_0}{\lambda} - \frac{w_0 - w_W}{\lambda}}{\lambda} = \frac{w_E - 2w_0 + w_W}{\lambda^2}$$

Similarly, the following expressions can be derived:

$$\frac{\partial^2 w}{\partial y^2} \simeq \frac{w_S - 2w_0 + w_N}{\lambda^2}$$

$$\frac{\partial^4 w}{\partial x^4} \simeq \frac{w_{WW} - 4w_W + 6w_0 - 4w_E + w_{EE}}{\lambda^4}$$

$$\frac{\partial^4 w}{\partial y^4} \simeq \frac{w_{SS} - 4w_S + 6w_0 - 4w_N + w_{NN}}{\lambda^4}$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} \simeq \frac{4w_0 - 2(w_N + w_S + w_E + w_W) + (w_{NW} + w_{NE} + w_{SW} + w_{SE})}{\lambda^4}$$

Substitute these expressions into the author's Eq. 2 the following difference equation at point "0" is obtained.

$$q\lambda^4 = (6D_x + 6D_y + 8H)w_0 - 4(D_y + H)(w_N + w_S) - 4(D_x + H)(w_E + w_W) + 2H(w_{NW} + w_{SW} + w_{NE} + w_{SE}) + D_y(w_{NN} + w_{SS}) + D_x(w_{WW} + w_{EE}) \quad \text{---(1)}$$

At each net point, an equation similar to the writer's Eq. 1 can be obtained. The number of the linear equations equals the number of net points in the net-work. By solving these linear equations simultaneously, the deflections at all the net points can be determined. If a finer net is used, the results will be more accurate.

The bending moments per unit length in both directions, Mx and My, can be found from the deflections just determined. For example, the bending moments at both ends of the bar OW in the writer's Fig. 2 are:

$$M_{x(OW)} = -D_x \frac{\partial^2 w}{\partial x^2} = -D_x \frac{w_E - 2w_0 + w_W}{\lambda^2} \quad \text{--- (2)}$$

$$M_{x(WO)} = -D_x \frac{\partial^2 w}{\partial x^2} = -D_x \frac{w_0 - 2w_W + w_{WW}}{\lambda^2}$$

Illustrative Example: Find the moments and deflections of a concrete filled steel-grid bridge floor, 9 ft wide, 12 ft long, continuous over two spans as shown in the writer's Fig. 3 due to the application of two 1 kip loads at points 8 and 8'. Each load is distributed uniformly over an area 18 inch square.

Given:

$$D_x = 23{,}300 \quad \text{Kip. inch}$$
$$D_y = 4{,}820 \quad \text{Kip. inch}$$
$$H = 14{,}360 \quad \text{Kip. inch}$$

Since both the edge and load conditions are symmetrical about the mid-support A-A, each half of the slab can be considered as fixed along A-A and simply supported along other three sides. By the use of the writer's Eq. 1, the following nine simultaneous equations can be set up.

$$\text{Pf}1,\ 0 = 283.62\,w_1 - 76.73\,w_4 - 150.65\,w_2 + 28.68\,w_5 + 4.82(-w_1 + w_4) + 23.30(-w_1 + w_5)$$

$$2,\ 0 = 283.62\,w_2 - 76.73\,w_5 - 150.65(w_3 + w_1) + 28.68(w_4 + w_6) + 4.82(-w_2 + w_8)$$

$$3,\ 0 = 283.62\,w_3 - 76.73\,w_6 - 150.65\,w_2 + 28.68\,w_5 + 4.82(-w_3 + w_9) + 23.30(w_1 + w_5)$$

$$4,\ 0 = 283.62\,w_4 - 76.73(w_1 + w_7) - 150.65\,w_5 + 28.68(w_2 + w_8) + 4.82\,w_4 + 23.30(-w_4 + w_7)$$

$$5,\ 0 = 283.62\,w_5 - 76.73(w_2 + w_8) - 150.65(w_6 + w_4) + 28.68(w_1 + w_3 + w_7 + w_9) + 4.82\,w_5$$

$$6,\ 0 = 283.62\,w_6 - 76.73(w_3 + w_9) - 150.65\,w_5 + 28.68(w_2 + w_8) + 4.82\,w_6 + 23.30(w_4 + w_8)$$

$$7,\ 0 = 283.62\,w_7 - 76.73(w_4 + w_7) - 150.65\,w_8 + 28.68(w_5 + w_5) + 4.82(w_1 + w_7) + 23.30(-w_7 + w_9)$$

$$8,\ \frac{\lambda^2}{1000} = 283.62\,w_8 - 76.73(w_5 + w_5) - 150.65(w_9 + w_7) + 28.68(w_4 + w_6 + w_4 + w_6) + 4.82(w_2 + w_8)$$

$$9,\ 0 = 283.62\,w_9 - 76.73(w_6 + w_6) - 150.65\,w_8 + 28.68(w_5 + w_5) + 4.82(w_3 + w_9) + 23.30(w_7 + w_9)$$

Where $\lambda = 18"$.

In forming the equations for the points near the boundaries, imaginary deflections at points outside the boundaries are required. The imaginary deflections exterior to the slab are symmetrical with that in the floor about the fixed edge. For simply supported edge, the corresponding deflections are equal in magnitude, but with opposite signs. The general procedure in forming and solving similar simultaneous equations is available.[3] The solutions, in inches, of the above set of equations are:

$$w_1 = 0.00055 \qquad w_6 = 0.00097$$
$$w_2 = 0.00071 \qquad w_7 = 0.00259$$
$$w_3 = 0.00032 \qquad w_8 = 0.00408$$
$$w_4 = 0.00146 \qquad w_9 = 0.00191$$
$$w_5 = 0.00201$$

The maximum negative moment at support A A, -Mx, the maximum positive moment at the center of the span, Mx, as well as the moment in the longitudinal direction, My, can be found by using equations similar to the writer's Eq. 2.

3. See, for example, "Analysis of Skew Plates" by V. P. Jensen, Univ. of Illinois Engineering Experiment Station Bulletin No. 332, 1941.

$$-M_x = -D_x \frac{W_E - 2W_o + W_W}{\lambda^2} = -23{,}300 \times \frac{2 \times 0.00191}{324} = -0.275 \text{ in-kips/in}$$

$$M_x = -23{,}300 \times \frac{(0.00259 - 2 \times 0.00408 + 0.00191)}{324} = +0.263 \text{ in-kips/in}$$

$$M_y = -4{,}820 \times \frac{(2 \times 0.00201 - 2 \times 0.00408)}{324} = +0.062 \text{ in-kips/in}$$

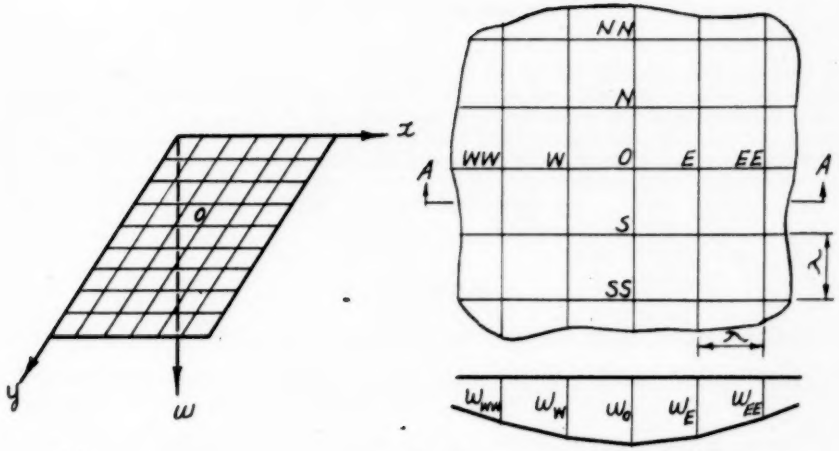


Fig. 1. Section A-A

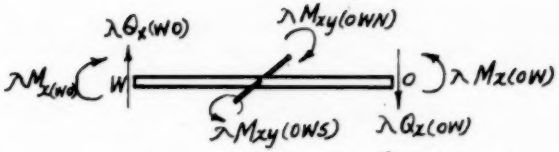


Fig. 2.

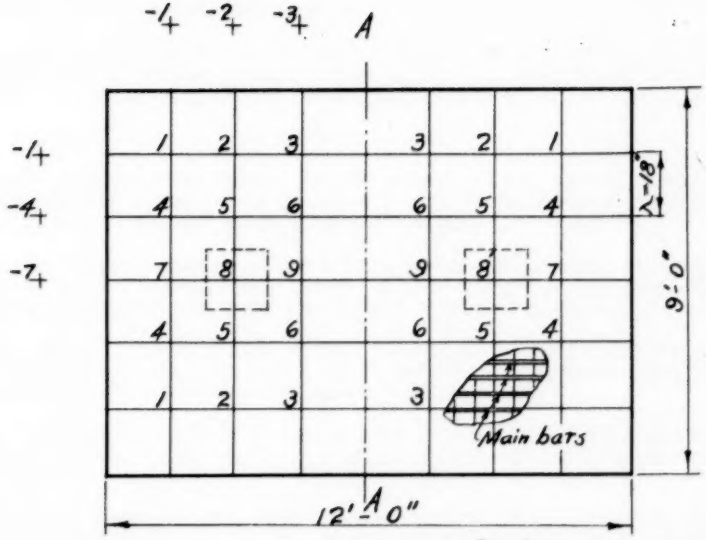


Fig. 3

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 80 (1954)

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c] 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

VOLUME 81 (1955)

JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)[c], 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)[c], 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)[c], 700(PO), 701(ST)[c].

c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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